International Conference on Mechanics and Ballistics
"VIII Okunev's Readings"
25-28 June 2013. St. Petersburg, Russia

Review of the Variational Principles in Different Fields of Physics

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The main ideas

1. Most part of physics is described by the variational or extremal principles for appropriate boundary conditions.
2. Most of these principles related to each other mathematically or via the analogy of mechanical, optical and wave phenomena.
3. Some of these principles can be related to quantum mechanics

Why is it necessary in mechanics?

Understanding of the universality of variational principles may be useful in the learning process of students.

Physicists about the principle of least action

- The general relativity could be developed from the single variational principle (***Einstein, A.*** *(1965). The Collection of scientific works. Moscow. Vol. 1, p. 524*).
- The principle of least action is a more universal law of nature than the law of conservation of energy and momentum, so it dominates above all reversible phenomena of physics (***Planck, M.*** *(1925). The physical essays. Moscow. p. 95*).
- There are two great generalizations of science: the principle of least action and the second law of thermodynamics *(**Eddington, A.** (2003). Space, time and gravitation. Moscow. p. 149).*
- The principle of least action principle lies at the core of much of contemporary theoretical physics *(**Moore, T. A.** (1996). In the entry on "least-action principle" in Macmillan Encyclopedia of Physics. New York. Vol. 2, pp. 840–842.*

There are two questions:

How the extremal principles are related to each other?

How these principles can be related to other laws of physics?

The general form of the variational principles (integral)

(1) Actual motion (state) of system differs from any possible motions (states) under the given boundary conditions that the variation of the functional (action) of the system is zero.

(2) The functional (action) is stationary, and it takes extremal value.

(3) The functional (action) is computed as an integral of the state function (Lagrangian) of the system. It may be the time integral, the coordinate integral, the volume integral, or the space-time integral.

$$(1)\ \delta S = 0,\ (2)\ S = \min\ (\max),\ (3)\ S = \int L dt\ (ds, dV, d\Omega)$$

What is the basis of the general form of the variational principles:

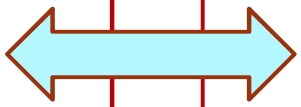
a) the convenience of calculations or b) the general nature of the phenomena?

Optical-mechanical analogy

Principle of least action (PLA)

Action (mass x velocity x coordinate) = min

δ Action = 0



Fermat's principle

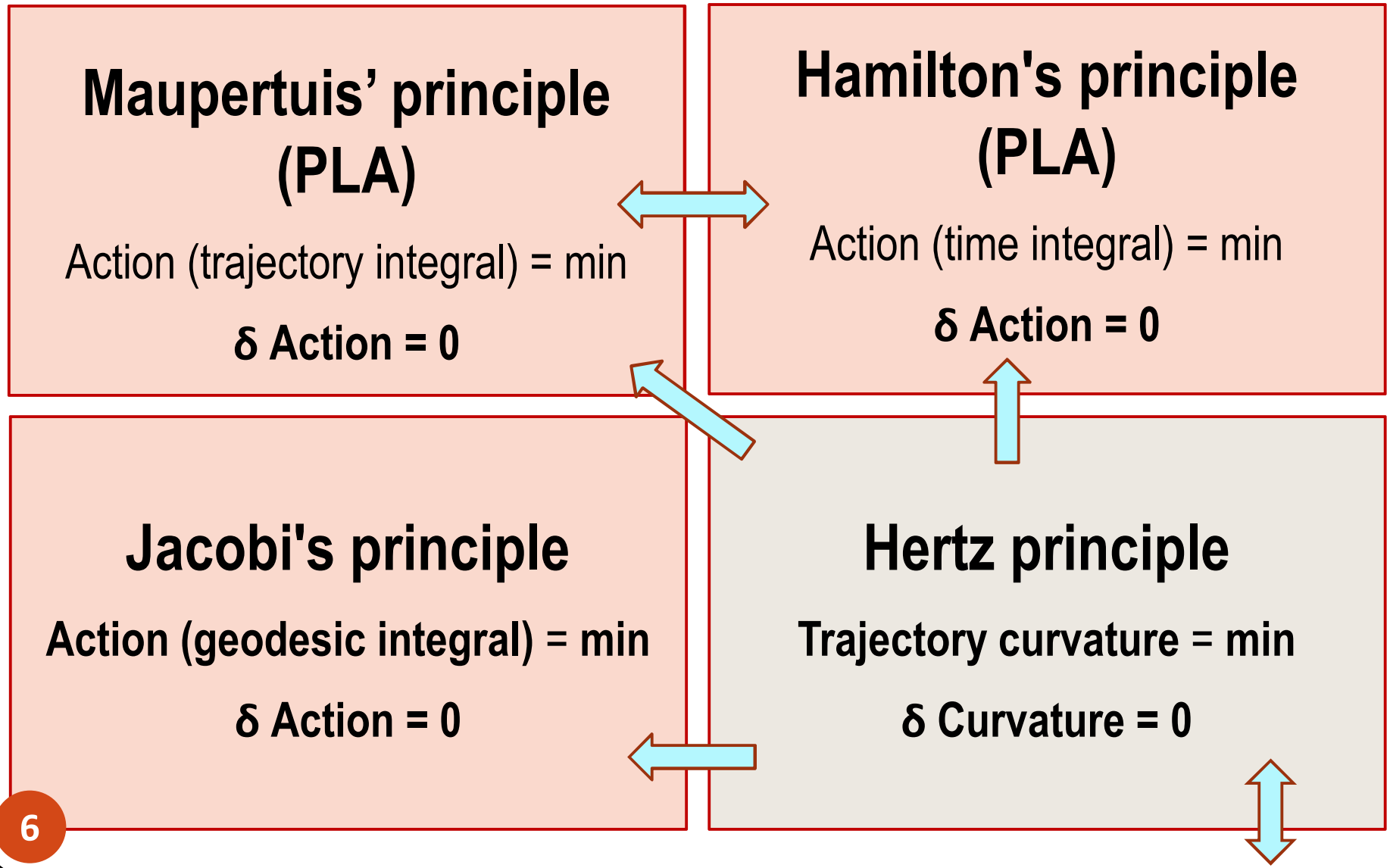
Time of ray of light = min

δ Time = 0

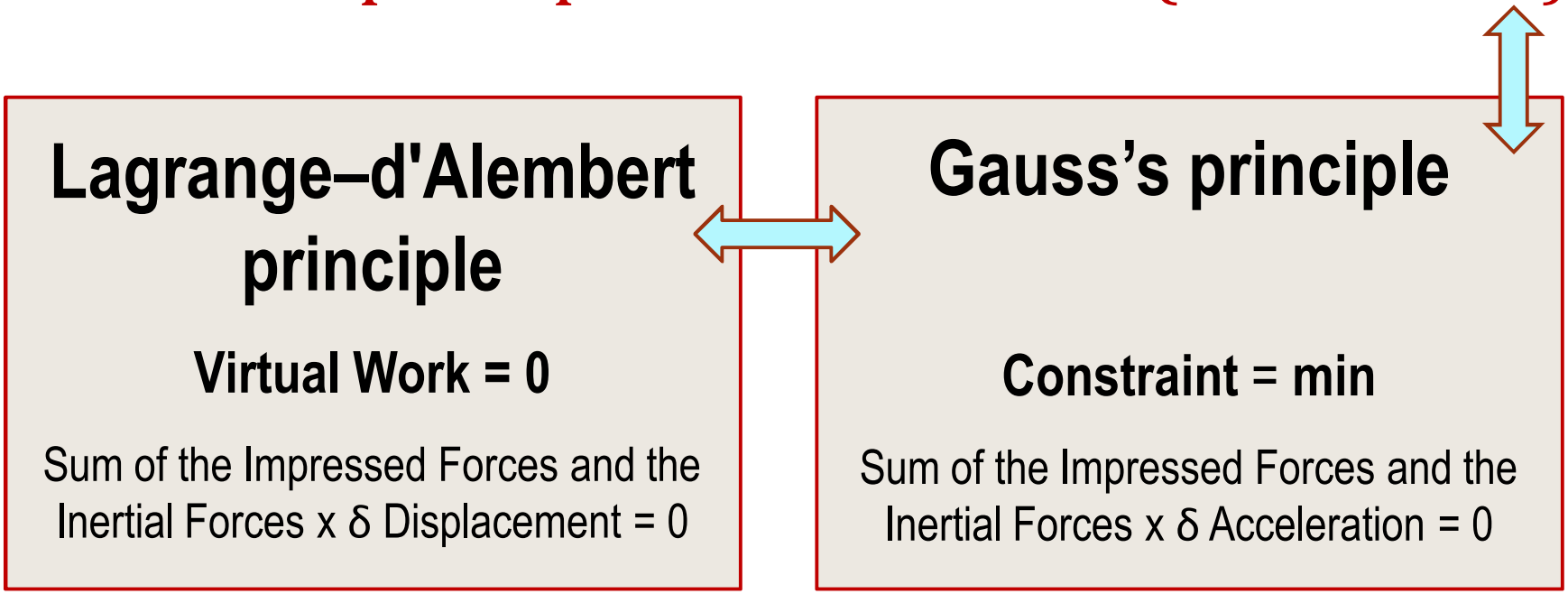
Why do body and light behave the same?

Why does nature saves the action (energy x time)?

Mechanical analogs of Fermat's principle



Variational principles of mechanics (differential)



The optical-mechanical analogy can be extended to the field replacing the Lagrangian by Lagrangian density

Variational principles for fields

PLA of electromagnetic (E/B) field

Field's action (volume integral or the 4-dimensional space-time integral) = min

δ Action = 0

PLA gravitational field

Field's action (volume integral or the 4-dimensional space-time integral) = min

δ Action = 0

PLA of charged particle in E/B field

Field's action + charge's action + charge and field interaction's action = min

δ Sum of actions = 0

PLA of charged particle in gravitational field

Field's action + particle's action + particle and field interaction's action = min

δ Sum of actions = 0

Towards quantum mechanics

Fermat's principle
motion of light rays

Hamilton's principle
motion of bodies

Huygens' principle
motion of waves

L. de Broglie,
E. Schrödinger

the Analogy

P. Dirac,
R. Feynman

Wave quantum mechanics

Quantum field's action (volume integral or the 4-dimensional space-time integral) = min

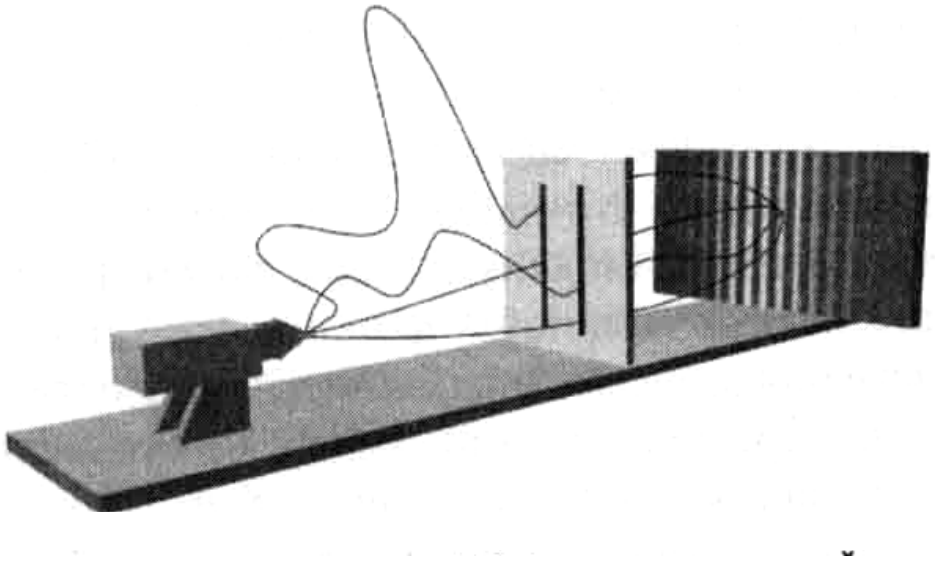
Path integral method

Absolute squared of the sum of probability amplitudes of all possible trajectories = max

$$p = |K|^2 = \max$$

$$K(b,a) = \sum_{\substack{\text{all possible} \\ \text{path} \\ \text{from a to b}}} \varphi,\ \varphi = const \cdot e^{jS/\hbar}$$

Feynman's path integral method



TWO-SLIT EXPERIMENT:

А) the particle passes through both slits at once as a wave (Huygens)

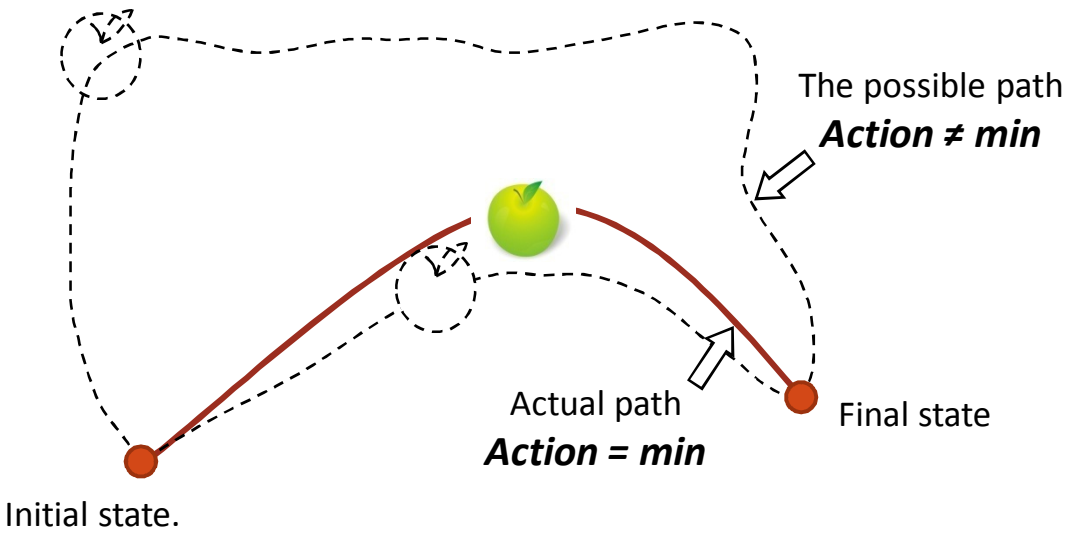
Б) the particle moves along all possible (virtual) paths, and each path has own phase or the quantum action (Feynman)

- The true path of the particle is the sum (interference) of the phases of all possible paths
- The phase or action of the true path is minimal, and its probability is maximal. Other ways exist, but they extinguish each other and are not observed
- Each particle flew not only in both slits simultaneously, but also to the Moon

What is the connection between quantum particles and classical objects?

Path Integral method for classical body

The Moon



We can imagine that :

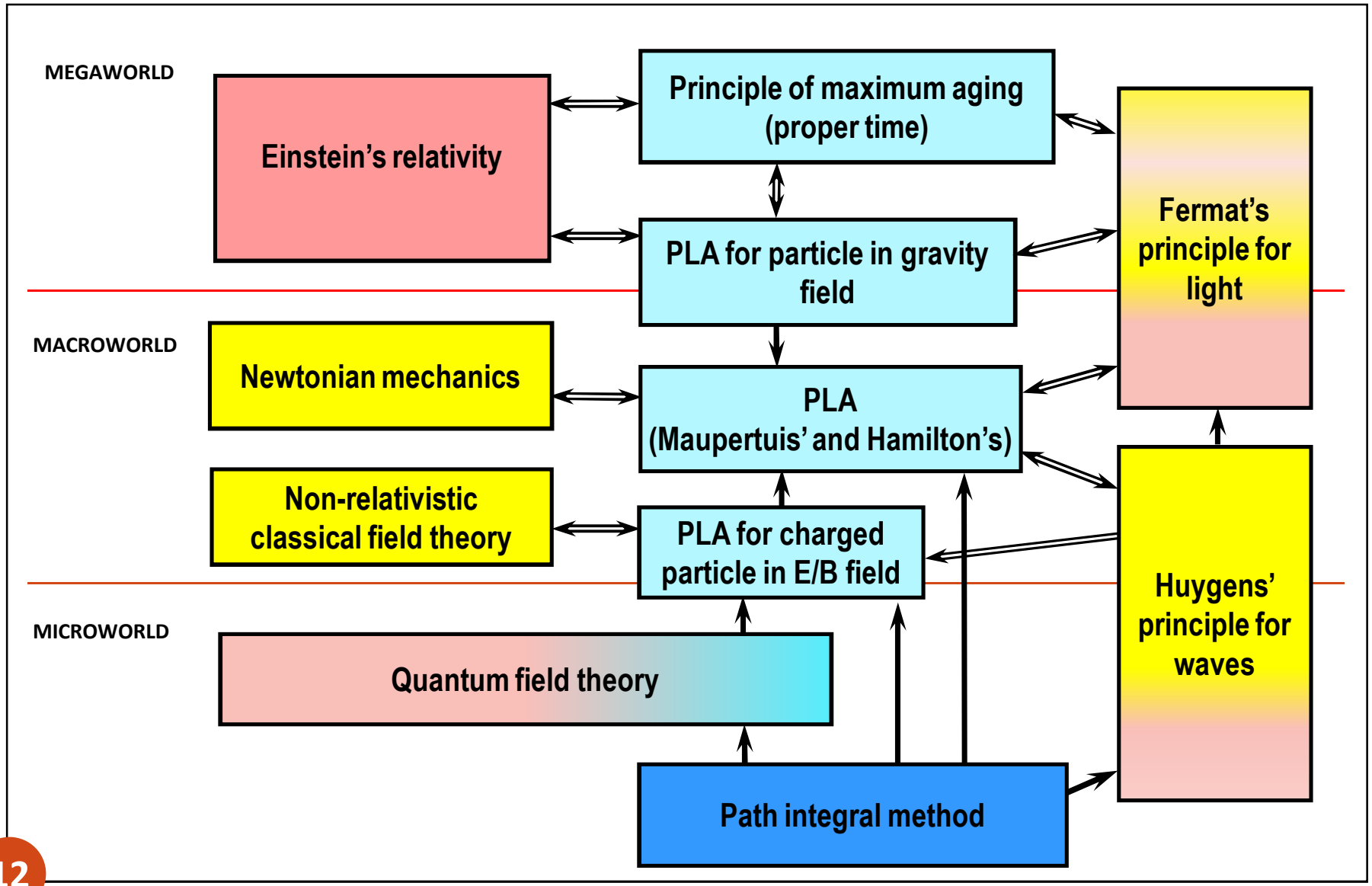
- the classical body behaves like a quantum particle, it flies along all the possible trajectories
- the true trajectory of the classical body is the sum of all possible trajectories (sum of their phases-actions)

The actual trajectory of the classical body can be calculated by several ways:

- through the forces of gravity and inertia (classical mechanics)
- through the potentials of the fields (field theory)
- through the geodesic lines in curved space-time (General Relativity)
- through the action (Hamilton's principle)

How do these methods of calculating are connected with the path integral method?

The relationship the variational principles (central column) with other principles of motion



The main conclusions

1. Most part of physics can be described in terms of variational or extremal principles for appropriate boundary conditions.
2. Most of these principles related to each other mathematically or via the analogy of mechanical, optical and wave phenomena. Most of them can be obtained through each other with different assumptions.
3. There is reason to believe that these principles are limiting cases of the quantum path integral method, and they can be summarized by the principle of maximum probability.

Why is it necessary in mechanics?

Understanding of the universality of variational principles may be useful in the learning process of students.

Related links

- Feynman, R. (1965). *The Character of Physical Law*. The M.I.T. Press.
- Feynman, R. P., & Hibbs, A. R. (1965). *Quantum mechanics and path integral*. New York: McGraw-Hill.
- Brizard, A. J. (2008). An Introduction to Lagrangian Mechanics. Singapore: World Scientific.
- Goldstein, H., Poole, C., & Safko, I. (2002). Classical Mechanics, 3rd edition. New York: Addison-Wesley.
- Polak, L. S. (Ed.) (1959). *Variational principles of mechanics*. Moscow: Fizmatgiz (in Russian).
- Polak, L. S. (2010). *Variational principles of mechanics: their development and application in physics*. Moscow (in Russian).
- http://www.eftaylor.com/leastaction.html - website of Edwin F. Taylor
- Terekhovich, V. E. (2013). Philosophical and methodological problems of the principle of least action, Ph. D. Dissertation, Saint-Petersburg State University, Russia (in Russian).
- Terekhovich, V. E. (2012). Probabilistic and Geometric Languages in the Context of the Principle of Least Action. *Philosophy of Sciences (Novosibirsk), 1*, 80-92. *arXiv preprint arXiv:1210.1176.*

Action is the key to whole physics!

THANK YOU

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ResearcherID: O-1251-2013

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